

Research on Parameterization of spacecraft trajectory collision probability based on Covariance Analysis

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Abstract: In order to avoid collision between on-orbit spacecraft and other space targets, when necessary, spacecraft need to take active orbit maneuver to avoid collision risk. With the increase of human space missions and the emergence of a large number of rocket arrows and their debris, invalid satellites and other space debris, the collision probability between space targets is increasing. At the same time, before the daily orbit adjustment or orbit maintenance of the spacecraft, it is also necessary to carry out collision early warning analysis to avoid collision during or after orbit control. To solve this kind of problem, it is necessary to study the influence of orbit control error on collision probability. In this paper, firstly, the propagation model of orbit control error is established based on the description function method of covariance analysis. Secondly, on the basis of the existing calculation methods of two-dimensional collision probability, the calculation method of three-dimensional collision probability of space target under nonlinear relative motion is further given. The calculation accuracy of collision probability is improved. Finally, the influence of orbit control error on collision probability is discussed. The simulation verification of the nonlinear method through specific collision events shows that the collision probability will increase with the increase of the influence of orbit control error on the propagation of position and velocity error, in which the introduction of thrust azimuth deviation has the greatest influence on the collision probability. The introduction of thrust magnitude deviation has the second influence on the collision probability, and the introduction of thrust high and low angle deviation has the least influence on the collision probability.

1. Introduction

With the increasing activities of human beings in space, the number of space targets in orbit, the appearance of a large number of rocket bodies and their debris, failed satellites and other space debris make the probability of collision between space targets increase continuously[1]. In order to avoid collision between spacecraft in orbit and other space targets, the spacecraft should take active orbit maneuver to avoid collision risk. At the same time, collision warning analysis is also needed before the spacecraft's daily orbit adjustment or orbit maintenance to avoid collision during or after orbit control [2]. To solve this kind of problem, it is necessary to study the influence of orbit control error on collision probability. Orbit error propagation analysis is an inevitable problem in target tracking and collision warning. As the number of space targets increases, the demand for target tracking and warning based on error analysis also increases.

The linearization of nonlinear systems and the quasi-linearization method, namely the covariance analysis describing function method, are proposed by the American Center for Analytical Sciences. CADET method can be used to analyze the statistical characteristics of linear or nonlinear systems with random errors. It has the characteristics of time saving, high precision and high reliability. Described the basic idea is to use function theory to statistical linearization system first, and then USES the covariance analysis, the mathematical model and the stochastic linear system state

variables mean and covariance propagation of differential equation, a single solution of the two differential equation can determine the statistical properties of the system state variables without the change process of system variable sampling arithmetic for hundreds of times. Therefore, the covariance analysis description function method USES less time than the sampling statistical calculation. At the same time, the covariance analysis describing function method not only saves time for linear system, but also gets exact solution, while for nonlinear system, it can get approximate solution. Gurfil [3] designed a robust missile guidance law based on covariance analysis description function method, and Wang et al. [4] used this method to study the missile landing point accuracy. At present, when the error propagation characteristics are studied by the covariance analysis method, orbital motion is often simplified as a two-body problem, which is convenient for the linearization of Taylor expansion in the later stage. The first is that the orbit model is relatively simple, and the second is that the Taylor expansion linearization has a large error for long-term forecasting problems. Therefore, the analysis of orbit control error propagation characteristics considering perturbation and control force needs to be further studied. Based on covariance analysis describing function method to establish the rail control error propagation model, secondly in the existing two-dimensional collision probability calculation method on the basis of the further nonlinear relative motion of space target is given and the calculation method of 3 d collision probability to improve the calculation precision of the collision probability, finally discussed the rail control error on the influence of the collision probability.

2. Orbit control error propagation model

General continuous time nonlinear systems differential equation:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, t) + \mathbf{G}(t)\mathbf{w}(t) + \mathbf{D}(t) \quad (1)$$

Where $\mathbf{f}(\mathbf{x}, t)$ is a nonlinear vector function of n -dimensional state variable $\mathbf{x}(t)$, and the meanings of other quantities are the same as those of linear systems.

Given that the estimated mean value and covariance of orbital states at the initial time are:

$$\mathbf{x}(0) \sim N(\bar{\mathbf{x}}_0, \mathbf{P}_0) \quad (2)$$

According to the covariance description function method, orbit prediction is to integrate the following propagation formula of estimated mean and covariance:

$$\begin{aligned} \dot{\bar{\mathbf{x}}}(t) &= \hat{\mathbf{f}}(\bar{\mathbf{x}}(t), t) = \mathbf{f}(\bar{\mathbf{x}}(t), t) + \\ &\frac{1}{2} \sum_{p=1}^n \sum_{q=1}^n P_{pq} \frac{\partial^2}{\partial x_p \partial x_q} \mathbf{f}(\mathbf{x}) |_{\mathbf{x}=\bar{\mathbf{x}}} \end{aligned} \quad (3)$$

$$\dot{\mathbf{P}} = \hat{\mathbf{F}}\mathbf{P} + \mathbf{P}\hat{\mathbf{F}}^T \quad (4)$$

$$\hat{\mathbf{F}}(\bar{\mathbf{x}}(t), t) = \frac{\partial \hat{\mathbf{f}}(\bar{\mathbf{x}}(t), t)}{\partial \bar{\mathbf{x}}(t)} \quad (5)$$

Definition:

$$\mathbf{M}_i = \frac{\partial^2}{\partial x_p \partial x_q} f_i(\mathbf{x}) |_{\mathbf{x}=\bar{\mathbf{x}}}, i = 1, 2, \dots, 6 \quad (6)$$

$$\begin{aligned} \mathbf{M}_i &= [f_i(\bar{x}_p + h_p, \bar{x}_q + h_q) - f_i(\bar{x}_p - h_p, \bar{x}_q + h_q) \\ &- f_i(\bar{x}_p + h_p, \bar{x}_q - h_q) + f_i(\bar{x}_p - h_p, \bar{x}_q - h_q)] / 4h_p h_q \end{aligned} \quad (7)$$

h_p and h_q are the steps for solving difference

The finite difference formula can also be used to solve the equation:

$$\hat{\mathbf{F}}(\bar{\mathbf{x}}(t), t) = \frac{\hat{\mathbf{f}}(\bar{x}_p + h_p) - \hat{\mathbf{f}}(\bar{x}_p - h_p)}{2h_p} \quad (8)$$

The covariance propagation matrix of each moment can be obtained by integrating equations (4) and (5).

The propagation model of orbit control error is established based on the quasi-linearization model:

$$\dot{\bar{x}}(t) = \hat{f}(\bar{x}(t), t) + \bar{w}(t) \quad (9)$$

$$\dot{P} = \hat{F}P + P\hat{F}^T + GQG^T \quad (10)$$

Where $w(t) \sim N(0, Q(t))$ the white noise of. According to the definition of $Q(t)$, $Q(t)$ is the spectral density matrix of the noise controlling input. Assuming that the control acceleration CA has deviations in magnitude and direction, its projection in the RSW coordinate system can be written as:

$$\mathbf{a}_c = \begin{bmatrix} (a_c + \delta a_c) \cos \alpha \sin \beta \\ (a_c + \delta a_c) \cos \alpha \cos \beta \\ (a_c + \delta a_c) \sin \alpha \end{bmatrix} \quad (11)$$

3. Calculation of collision probability of a space object with nonlinear relative motions

In general, due to the large relative velocity between the two targets, the joint sphere will quickly pass through the error ellipsoid, and the motion between the two can be approximately linear, so the area swept by the sphere in a certain period of time is a straight cylinder, as shown in figure 1. In the case of linear relative motion, the closest distance occurs in the plane perpendicular to the direction of the relative velocity vector, and only the state of the rendezvous time can be taken into account. When both the joint error ellipsoid and the joint sphere are projected into the meeting plane perpendicular to the relative velocity vector, the calculation of collision probability is transformed from three-dimensional integral to two-dimensional.

However, in some cases, if the relative velocity between two objects is small, the joint sphere will stay in the error ellipsoid for a long time, and the assumption of linear relative motion is no longer valid. What the joint sphere sweeps during this period is a curved cylinder, as shown in FIG. 2. The probability of collision becomes a function of time, so the probability of collision between the two objects is equal to the sum of the components of the probability of collision over this period of time. Obviously, FIG. 1 is a special case of FIG. 2, that is, the nonlinear collision probability calculation method is also used for linear relative motion.

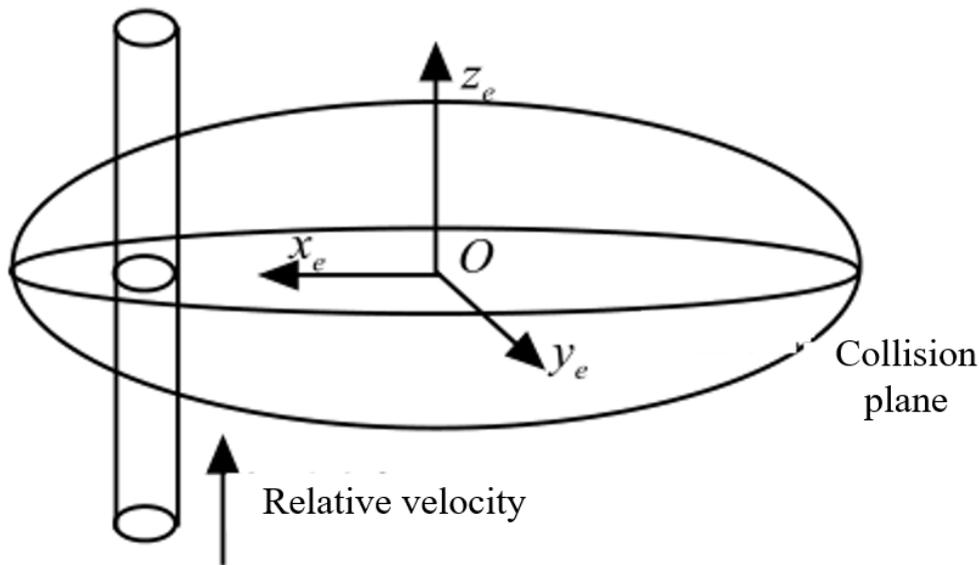


Fig 1. Geometric intersection of linear relative motion.

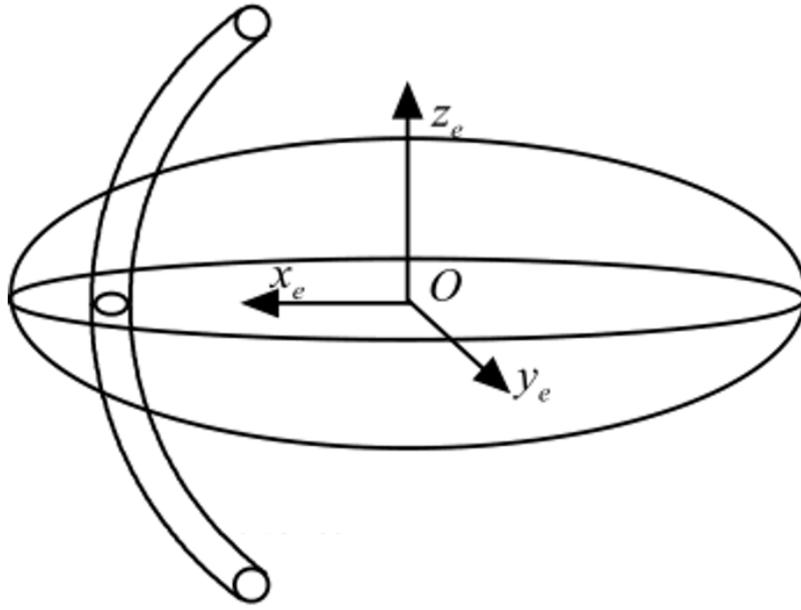


Fig 2. Geometric intersection of nonlinear relative motion.

For nonlinear relative motion, the integral volume is no longer an infinitely long cylinder. The calculation of nonlinear relative motion collision probability has the following characteristics:

- 1) The input parameters are the same as the calculation method of linear relative motion collision probability;
- 2) In the whole meeting process, the position error PDF can change with time, that is, the shape, size and direction of the error ellipsoid are variable;
- 3) The cross section of the space target on the interface can change with time.

According to the method of statistical orbit determination and error propagation theory, the position error of a space target at a certain moment can be described by three-dimensional Gaussian distribution. If the error distribution of two targets is not related, then the relative position error between them is equal to the sum of the position error of the two targets, forming a joint error ellipsoid. At the same time, the geometric size of the two objects is described by the equivalent radius, and the combined sphere is obtained by adding. If the center of the error ellipsoid is taken as the stationary origin, then the combined sphere will sweep over a certain volume in accordance with the relative motion velocity in a certain time. The collision probability is actually equal to the integral of the joint position error probability density function in this space region:

$$P = \frac{1}{\sqrt{(2\pi)^3 |C|}} \iiint_V \exp \left\{ -\frac{1}{2} [(\mathbf{rho}(t) - \mathbf{rho}_0)^T \mathbf{C}^{-1} (\mathbf{rho}(t) - \mathbf{rho}_0)] \right\} dx dy dz \quad (12)$$

Considering the collision time interval joint sphere over a period of curved cylinder, we put the curved cylinder divided into several small part, to a small part of them, if the time interval is small enough, can put the approximately as a linear motion, then the total collision probability is equal to the error probability density function in each period of small straight column inner integral sum.

So the total collision probability is equal to the sum of the components of the collision probability that you get for each fraction:

$$P_c = \sum P_i \quad (13)$$

4. Influence of orbit control error on collision probability

Linear relative motion can be regarded as a special case of nonlinear relative motion, so the nonlinear method can be verified by specific collision events. In order to analyze the propagation characteristics of the control force size and direction deviation on the orbit error and conduct subsequent collision probability research on this basis, the above methods can be verified through

specific collision events, such as the collision event between Chinese and American space debris at an altitude of 885km on January 17, 2005. Given the latest TLE before the collision of two targets, the SGP4/SDP4 orbital prediction model can be used to obtain the position and velocity vector of the rendezvous time (2005-01-1702:14:37.00UTCG) in the ECI coordinate system, as shown in Table 1.

Table 1. The position and velocity coordinates of the two pieces at the time of intersection.

	X/km	Y/km	Z/km	Vx/(km/s)	Vy/(km/s)	Vz/(km/s)
07219	-150.80	1182.68	-7142.2-77556	7.29	-1.13	-0.35
26207	-150.62	1182.02	-7142.2-78799	5.92	4.35	0.64

In the case of linear relative motion, first met using plane two-dimensional method of collision probability calculation, the result of 0.000678. Then calculated by the nonlinear method mentioned above, the result is shown in figure 3, as the integral time, the largest joint sphere began to enter the error ellipsoid, nonlinear method of collision probability value by zero began to increase, in the end when the ball left the error ellipsoid, collision probability value no longer increases, the result is 46.8410, compared with 2 d results error is less than 1%. This shows that the nonlinear method is compatible with linear relative motion, and also proves the nonlinear collision given in this paper.

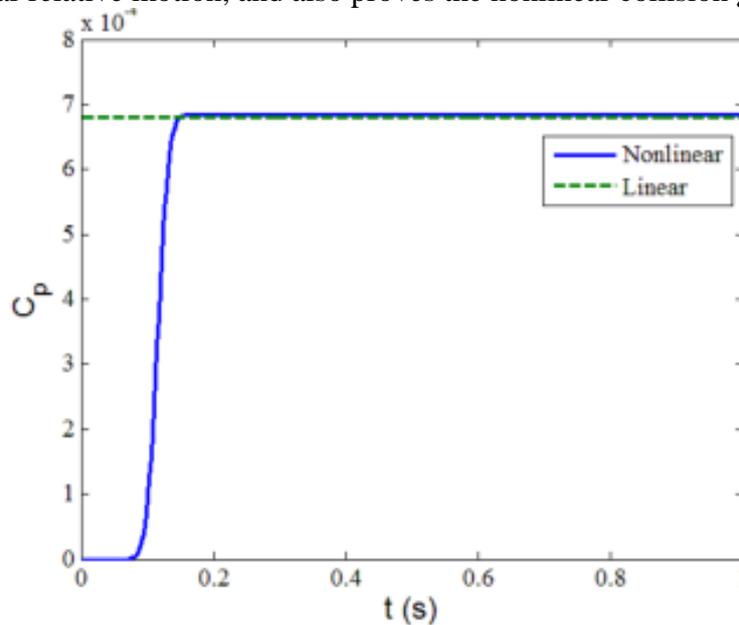


Fig 3. The probability of collision in linear relative motion.

5. Conclusion

In this paper, the orbit control error propagation on the influence of the collision probability problem is described, based on the covariance analysis describing function method to establish a track control error propagation model, the existing two-dimensional collision probability calculation method on the basis of the further nonlinear relative motion of space target is given and the calculation method of 3 d collision probability to improve the calculation precision of the collision probability, finally discussed the rail control error on the influence of the collision probability. The simulation results show that the impact probability will increase as the influence of the orbit control error on the propagation of position and velocity error increases, and the impact probability will be influenced the most by the introduction of thrust azimuth deviation, followed by the impact probability by the introduction of thrust magnitude deviation, and the impact probability will be influenced the least by the introduction of thrust high and low Angle deviation.

On the basis of this method, the future research work can be carried out in two directions: one is to combine the method with the decision algorithm in specific scenarios, and the other is to improve the computational efficiency of the method.

(1) Application scenario. A very typical application scenario is airport traffic control. For each flight in and out of the airport, according to its historical flight data, the corresponding expected trajectory and its uncertainty in all directions can be analyzed. Thus, this algorithm can be used to analyze whether each flight has the risk of interfering with normal work or even collision, which can be used as one of the bases of flight scheduling and route planning. If the throughput of the airport is small, the risk can be avoided simply by extending the interval between the two flights, but with the rapid increase in the number of air passengers, the density of flights and routes will inevitably increase, which requires the airport to carry out more efficient and safe planning from the perspective of risk control. In this paper, the algorithm combined with dichotomy can quickly iteratively calculate the interval time between two flights corresponding to a given allowable risk. With the further development and popularization of aviation technology, there may be busy unmanned air bus routes in the future, so it is more necessary to carry out collision risk assessment combined with track uncertainty to ensure flight safety.

(2) Calculation efficiency. If the application scenario requires high real-time performance, such as the on-line real-time route planning of aircraft, the computational efficiency of the improved method should be considered.

References

- [1] Geller D K. Linear Covariance Techniques for Orbital Rendezvous Analysis and Autonomous Onboard Mission Planning [J]. *Journal of Guidance Control, and Dynamics*, 2006, 29(6):1404~1410.
- [2] Geller D K, Rose M B, Woffinden D C. Event Triggers in Linear Covariance Analysis with Applications to Orbital Rendezvous [J]. *Journal of Guidance Control, and Dynamics*, 2009, 32(1): 102~111.
- [3] Gurfil P. Robust Zero Miss Distance Guidance for Missiles with Parametric Uncertainties [C]. *American Control Conference, IEEE*, 2001: 3364~3369.
- [4] Wang X, Xie H, Simulation of Covariance Analysis Describing Equation Technique (CADET) in missile hit probability calculation [C]. *6th International Conference on Natural Computation, IEEE*, 2010: 4282~4285.